8.0 Exponential Function

A Exponents Laws

The following relations are called *exponent laws*:

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn} \qquad a^{-n} = \frac{1}{a^n}$$

Note. If the exponent is a *rational number* then:

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$, q is a positive integer

B Definition

Exponential function $y = f(x)$ is defined by:

$$y = f(x) = b^x$$

where:

- b is the *base*
- x is the *argument*
- y is the *value* of the exponential function

Note. For each value of the base b a different exponential function is defined.

Ex 1. Find the *base* of the following exponential functions.

a) $y = 2^{3x}$

b) $y = \dfrac{1}{10^x}$

c) $y = 3^{\frac{x}{2}}$

d) $y = 2^{-3x}$

e) $y = \sqrt{3^x}$

C Restrictions

To avoid complex number values, the base b is positive: $b > 0$.
Indeed, if the base is negative, then the value of the exponential function is a complex number.

For example: $(-4)^{\frac{1}{2}} = \sqrt{-4} = \pm 2i$.

In order the exponential function to be a one-to-one function, the base must not be one: $b \neq 1$.

Indeed, if the base is one, then $1^x = 1$, which is not a one-to-one function and does not have an inverse function.

So, in conclusion the restrictions are:

$$b > 0, b \neq 1 \text{ or } b \in (0,1) \cup (1,\infty)$$

Ex 2. Determine if the exponential function is well defined (satisfies the restrictions).

a) $y = (-2)^x$

b) $y = -2^x$

c) $y = -1^x$

d) $y = \dfrac{1}{(-2)^x}$

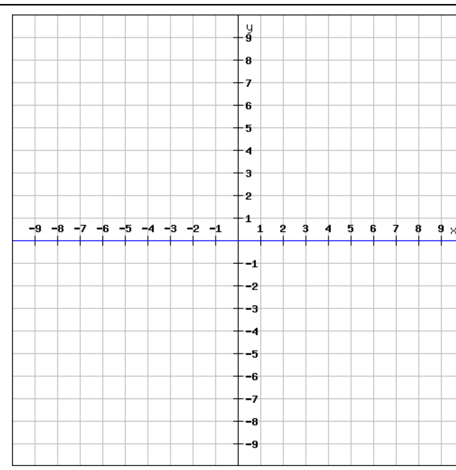
D The Graph of the Exponential Function

Ex 3. Graph on the same grid.

a) $y = 2^x$

b) $y = \left(\dfrac{1}{2}\right)^x$

Note. $y = \left(\dfrac{1}{2}\right)^x = 2^{-x}$, so the graphs are symmetric with respect to the y-axis.



E Characteristics of the Exponential Function

Ex 4. Use the graphs obtained at Ex 3 to find the following characteristics of the exponential function.

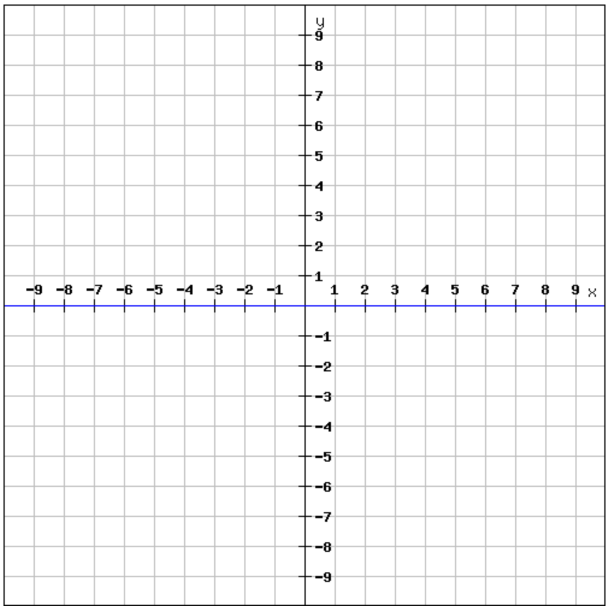
- Domain:
- Range:
- x-intercepts
- y-intercepts:
- Increasing/Decreasing:
- Horizontal Asymptotes:
- Vertical Asymptotes:
- Continuity:
- One-to-one:
- Key Points:

F Transformations

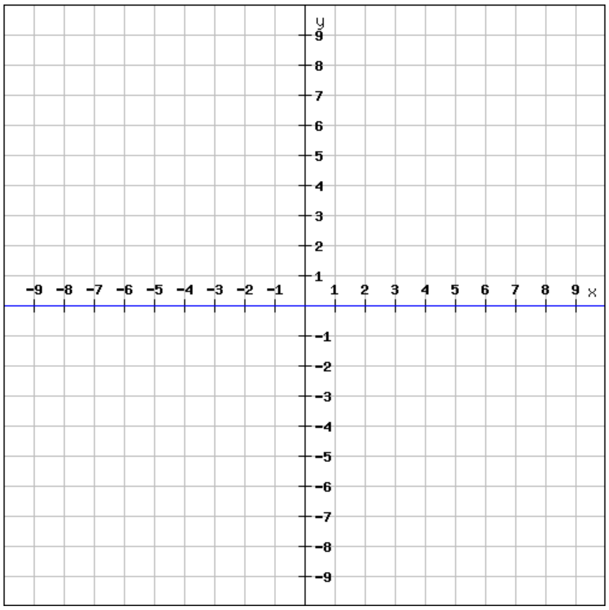
$$g(x) = A\,b^{B(x-C)} + D$$

Note. The equation of the horizontal asymptote is: $y = D$

Ex 5. Use transformations to graph the following exponential functions.



a) $y = -3^{-x+1} + 9$



b) $y = -4 + 2 \cdot 0.5^{x+2}$

G One-to-one property

The exponential function is a one-to-one function. Therefore:

$$b^x = b^y \Leftrightarrow x = y$$

